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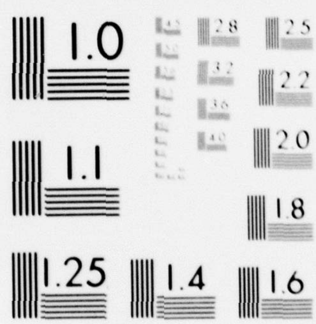

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FINAL SCIENTIFIC REPORT FOR GRANT AFOSR-73-2504

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TITLE: Nonlinear Programming Global Optimization Techniques

PRINCIPAL INVESTIGATORS: James E. Falk, Anthony V. Fiacco, Garth P. McCormick

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ABSTRACT

This report contains: a description of the state of affairs in global optimization which existed at the inception of this grant from the user point of view and also available algorithms; the results obtained in the five-year grant period for (a) separable optimization problems, (b) factorable problems, (c) infinitely constrained problems, and (d) other specially structured problems; a brief summary of the direct application of this work to Department of Defense planning studies. The basic result of this research is that rigorous, efficient, implementable algorithms are now available for obtaining global (as opposed to strictly local) optimizers to nonlinear programming problems defined by explicit functional relationships and having bounded variables.

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1. Introduction

The problem to which the research in this grant was addressed was that of finding global solutions to nonlinear optimization problems of the form:

$$\min_{x \in E^n} f(x)$$

subject to the restrictions that $g_i(x) \geq 0$ for $i = 1,\ldots,m$. For convex programming problems, local solutions to the problem are global solutions and hence no difficulty exists. But for the more general class of problems which are not-necessarily-convex (nonconvex) such problems may have local solutions which are not global solutions. Users of nonlinear programming algorithms are thus confronted with the possibility that the numbers printed out by computer coded algorithms are not the true solutions to their problem. Most general nonlinear programming algorithms determine (at best) a local solution of the given problem (i.e., a solution point that is optimal only in a neighborhood or vicinity of the point).

In 1973 when research began on this grant, users of nonlinear programming algorithms were usually advised to handle this problem by starting from different points. If the algorithms produced the same solution point, then it "probably" was the only local (and hence global) solution. The state of the art in algorithms which attempted to deal with the local vs. global problem was in disarray. In a survey of the multitude of ideas for dealing with the difficulty (McCormick [1]) it was found that except for a few papers (notably Falk and Soland [2] and Beale and Tomlin [3]) these ideas were unimplementable, or were computationally unacceptable because of the effort involved. As an example of the former it was suggested that the convex envelope of the function $f(x)$ be minimized in the convex hull of the feasible region. The well-known "small grid" approach is an example of the latter.

The research in the five-year period of this grant has been directed toward developing efficient, implementable, rigorous algorithms for guaranteeing the attaining of global solutions to nonconvex programming problems. This research has been successful in several areas. In Section 2 is a summary of the research on separable programming problems. Section 3 contains the results obtained on the very general class of factorable programming problems. Section 4 discusses infinitely constrained problems and in Section 5 are results for other structured problems. Some of the work has been applied to Department of Defense planning models, and this is taken up briefly in Section 6. Work remaining to be done, mostly in the area of implicitly defined problems is described in Section 7.

Papers supported by the grant are indicated with an asterisk in the reference section at the end.

2. Separable Programming

The Falk-Soland algorithm for separable, non-convex programs [2] was later extended by Soland [4] to apply the underlying branch and bound methodology directly to each of the functions defining the constraints. A parallel effort was carried out by Falk [5] who addressed piecewise linear separable problems, and effectively handled the constraints via a Lagrangian, or dual problem. The latter approach was shown by Greenberg [6] to yield sharper bounds than any method which took convex envelopes of the constraining function individually. In a later paper, Falk [7] extended Greenberg's results, and characterized problems wherein the bounds obtained were identical.

The work on the piecewise linear problems has proven to be efficient and practical, and a number of the applications addressed later were effectively solved via this method. Hoffman [8] coded a reliable version of the method during the contract period, which was later reprogrammed and extended by Grotte [9] of IDA.

Al-Khayyal [10] has obtained an efficient extension of the original Falk-Soland method as applied to "Bi-Convex Programs." This problem may be considered as more general than a separable problem, and a special case of the factorable problem addressed in Section 3.

Recent developments beyond the contract period are being made in the solution of separable equilibrium problems, and in multi-level optimization problems.

3. Factorable Programming

Almost all functions used in nonlinear programming for practical problems are factorable functions, i.e., functions of n variables which are complicated compositions of transformed sums and products of transformations of transformed sums and products of functions of a single variable. Branch and bound algorithms for obtaining global solutions usually require that convex underestimating problems be available for non-convex problems restricted to hyperrectangles. As was mentioned in the previous section, this is fairly easy to do for separable problems. In McCormick 1976 [11] is a procedure for doing this when the problem functions are factorable. This paper also contains splitting rules to use in a branch and bound scheme for obtaining global solutions. Convergence to a global optimizer was proved.

In addition to the use of convex underestimating problems, another tool in algorithms for non-convex programming is the ability to verify that a local minimizer is a global minimizer in a convex compact set. For twice continuously differentiable functions minimized subject to bounded variables, the theory for verifying that a local unconstrained minimizer existed in a hyperrectangle of bounds and placing tight bounds on its location was given in Mancini and McCormick [12]. A paper developing the practical computational aspects of this theory was also done (Mancini and McCormick [13]) and will be published in Operations Research in 1979. This was illustrated by an example of the optimal design of a refinery.

The generalization of this verification principle for general non-convex programs was just completed and the paper has been submitted to Math. of O.R. [14].

These four papers complete the requirements for an algorithm to obtain global solutions to factorable programming problems with bounded variables.

4. Infinitely Constrained Problems

A preliminary study done by Bracken and McGill [15] involving optimization problems with optimization problems defining the constraints led to a formulation of a general mathematical program with an infinite number of constraints. An effective solution procedure, based on a relaxation of the constraints, was developed and published by Blankenship and Falk [16].

The basic algorithm involves the solution of a sequence of finitely constrained problems, with a check for optimality at each step. The check itself may involve the solution of an optimization problem (as in the Bracken and McGill application), or may be trivial (as in certain scheduling problems, e.g., [17]).

Convergence was established under very general conditions, and hence applies to non-convex, as well as convex problems.

The general context of the problem and its solution procedure has lent itself to a wide variety of applications (well beyond the Bracken-McGill model), some of which are mentioned in Section 7. Certain varieties of "max-min" problems are especially well treated with this method. And the coupling of this method with the non-convex methodology developed here at The George Washington University has resulted in a practical tool that can be applied to previously unsolvable problems (see, e.g., [18]).

5. Algorithms for Structured Non-separable Problems

The principal difficulty in non-convex programming stems from the necessity of compiling global estimates of a function's behavior based on relatively few point calculations. Separability alleviates the situation considerably, since it, in effect, allows one to treat problems in several variables by using function values of several functions of single variables. If these functions have special structure (e.g., convex, or concave), the Falk-Soland algorithm may be directly employed. Otherwise, piecewise linear approximations may be employed as in [5].

The problems wherein separability is difficult or expensive to obtain, only specially structured problems may be treated. One such problem that was solved under the grant was the problem of minimizing a concave function over a linear polyhedron. Falk and Hoffman [19] were able to compute convex envelopes of the objective function over successively tighter relaxations of the constraint region, thereby forcing convergence when the relaxations agreed with the feasible region. The relaxations were tightened by adding original constraints successively, so that finite convergence was guaranteed.

Several applications, including a selective strike model for bomber deployment are discussed in Hoffman [20].

Another class of problems which could be separated, at the expense of additional variables and constraints, was treated directly by Al-Khayyal [10]. These problems are characterized by the presence of terms of the form $x_i y_i$, and appear to be linear in a set of variables if all other variables are held constant, and conversely. The basic algorithm of Falk-Soland [2] was generalized, with appropriate modifications, and convergence proofs obtained.

Immediate applications to "linear complementarity" problems were obtained.

Finally, an algorithm for producing global solutions to "signomial" programs" was first published in [21].

6. Application of Non-convex Programming

In addition to an unusually high number of methodological results, this contract has also produced an exceptional list of direct applications of the theory.

A tool (TAC CONTENDER) for assessing figures of merit for developmental aircraft was generated by Air Force personnel, and used to support policy decisions. There were at least two major flaws in the model, one resulting from convergence to a local, as opposed to global, solution. These are discussed in [22].

A weapons allocation model first developed by Shere and Wingate [23] was significantly extended and solved by Falk and Hoffman [18].

Methodology relating to non-convex programming as well as the infinitely constrained work was used to generate solutions. It is important to note that a model of this type of complexity could not have been solved with the tools available before the grant period.

An alternative to the aforementioned TAC CONTENDER was published by Bracken, Falk, and Karr [24]. While, as opposed to TAC CONTENDER, the new model solved sequential games exactly, it could only apply to games of relatively short duration. The philosophy of infinitely constrained programming was used to accelerate convergence.

Additional comments on the multi-stage games are contained in the IDA paper [25].

Another application of the combined non-convex/infinitely constrained methodology was obtained in a project scheduling context [17] and [26]. Here, the problem can be cast in our optimization format, but with an excessively large number of constraints (the number of variables, factorial). Infinitely constrained programming was shown to be capable of generating solutions to an apparently unsolvable problem. Non-convex cost functions could easily be handled.

A two-strike nuclear exchange model was first formulated and solved by Bracken, Falk and Miercourt [27]. The intrinsic non-convex nature of the problem was easily handled by techniques developed in this study.

7. Summary

In Sections 1-6 the main results are listed in detail. Section 8 contains the references for those sections. The papers credited to the Grant are denoted with an asterisk (*). Other byproducts of the research sponsored by the Grant are listed in Section 9. These also are contained preliminary reports eventually published in the open literature and referenced in Section 8.

Work remains to be done in the area of implicitly defined problems.

An unusually large body of theoretical results of an applicable nature were generated by this grant. In addition to effectively building a workable set of algorithms for the non-convex problem, other spin-offs were found in the fields of infinitely constrained programming, and in multi-stage games.

The theoretical results obtained in this study were directly applied to a wide variety of models with applications of specific interest to the Air Force.

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This report summarizes the results obtained in the area of optimization with non-convex functions. The difficulty is in determining when a local minimum is also a global minimum. The results can be classified into four areas: ~~(1)~~ separable problems, ~~(2)~~ factorable problems, ~~(3)~~ infinitely constrained problems, and ~~(4)~~ other specially structured problems.

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